# SEKOLAH-SEKOLAH MENENGAH ZON A KUCHING 

## PEPERIKSAAN PERCUBAAN SIJIL PELAJARAN MALAYSIA 2009

## MATEMATIK TAMBAHAN

Kertas 2
Dua jam tiga puluh minit

## JANGAN BUKA KERTAS SOALAN INI SEHINGGA DIBERITAHU

1. This question paper consists of three sections: Section $\boldsymbol{A}, \operatorname{Section} \boldsymbol{B}$ and Section $\boldsymbol{C}$.
2. Answer all question in $\operatorname{Section} \boldsymbol{A}$, four questions from $\operatorname{Section} \boldsymbol{B}$ and $\mathbf{t w o}$ questions from Section C.
3. Give only one answer / solution to each question..
4. Show your working. It may help you to get marks.
5. The diagram in the questions provided are not drawn to scale unless stated.
6. The marks allocated for each question and sub-part of a question are shown in brackets..
7. A list of formulae is provided on pages 2 to 3 .
8. A booklet of four-figure mathematical tables is provided.
9. You may use a non-programmable scientific calculator.

Kertas soalan ini mengandungi $\mathbf{1 1}$ halaman bercetak

The following formulae may be helpful in answering the questions. The symbols given are the ones commonly used.
$1 x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}$
$2 \quad a^{m} \times a^{n}=a^{m+n}$
$3 \quad a^{m} \div a^{n}=a^{m-n}$
$4 \quad\left(a^{m}\right)^{n}=a^{m n}$
$5 \quad \log _{a} m n=\log _{a} m+\log _{a} n$
$6 \quad \log _{a} \frac{m}{n}=\log _{a} m-\log _{a} n$
$7 \quad \log _{a} m^{n}=n \log _{a} m$

## ALGEBRA

$8 \quad \log _{a} b=\frac{\log _{c} b}{\log _{c} a}$
$9 \quad T_{n}=a+(n-1) d$
$10 \quad S_{n}=\frac{n}{2}[2 a+(n-1) d]$
$11 T_{n}=a r^{n-1}$
$12 S_{n}=\frac{a\left(r^{n}-1\right)}{r-1}=\frac{a\left(1-r^{n}\right)}{1-r},(r \neq 1)$
$13 \quad S_{\infty}=\frac{a}{1-r},|r|<1$

## CALCULUS

$1 y=u v, \frac{d y}{d x}=u \frac{d v}{d x}+v \frac{d u}{d x}$
$2 y=\frac{u}{v}, \frac{d y}{d x}=\frac{v \frac{d u}{d x}-u \frac{d v}{d x}}{v^{2}}$,
$3 \frac{d y}{d x}=\frac{d y}{d u} \times \frac{d u}{d x}$

4 Area under a curve

$$
\begin{aligned}
& =\int_{a}^{b} y d x \text { or } \\
& =\int_{a}^{b} x d y
\end{aligned}
$$

5 Volume generated

$$
\begin{aligned}
& =\int_{a}^{b} \pi y^{2} d x \text { or } \\
& =\int_{a}^{b} \pi x^{2} d y
\end{aligned}
$$

## GEOM ETRY

1 Distance $=\sqrt{\left(x_{1}-x_{2}\right)^{2}+\left(y_{1}-y_{2}\right)^{2}}$
2 Midpoint
$(x, y)=\left(\frac{x_{1}+x_{2}}{2}, \frac{y_{1}+y_{2}}{2}\right)$
$3|r|=\sqrt{x^{2}+y^{2}}$
$4 \hat{r}=\frac{x i+y j}{\sqrt{x^{2}+y^{2}}}$

5 A point dividing a segment of a line

$$
(x, y)=\left(\frac{n x_{1}+m x_{2}}{m+n}, \frac{n y_{1}+m y_{2}}{m+n}\right)
$$

6. Area of triangle =

$$
\frac{1}{2}\left|\left(x_{1} y_{2}+x_{2} y_{3}+x_{3} y_{1}\right)-\left(x_{2} y_{1}+x_{3} y_{2}+x_{1} y_{3}\right)\right|
$$

## STATISTICS

$1 \bar{x}=\frac{\sum x}{N}$
$7 \bar{I}=\frac{\sum w_{1} I_{1}}{\sum w_{1}}$
$2 \bar{x}=\frac{\sum f x}{\sum f}$
$8 \quad{ }^{n} P_{r}=\frac{n!}{(n-r)!}$
$9 \quad{ }^{n} C_{r}=\frac{n!}{(n-r)!r!}$
$3 \sigma=\sqrt{\frac{\sum(x-\bar{x})^{2}}{N}}=\sqrt{\frac{\sum x^{2}}{N}-\bar{x}^{2}}$
$10 \quad P(A \cup B)=P(A)+P(B)-P(A \cap B)$
$4 \quad \sigma=\sqrt{\frac{\sum f(x-\bar{x})^{2}}{\sum f}}=\sqrt{\frac{\sum f x^{2}}{\sum f}-\bar{x}^{2}}$
$11 \quad P(X=r)={ }^{n} C_{r} p^{r} q^{n-r}, p+q=1$
12 Mean $\mu=n p$
$5 m=L+\left[\frac{\frac{1}{2} N-F}{f_{m}}\right] C$
$13 \quad \sigma=\sqrt{n p q}$
$14 \mathrm{z}=\frac{x-\mu}{\sigma}$
$6 \quad I=\frac{Q_{1}}{Q_{0}} \times 100$

## TRIGONOMETRY

1 Arc length, $s=r \theta$
2 Area of sector, $A=\frac{1}{2} r^{2} \theta$
$3 \sin ^{2} A+\cos ^{2} A=1$
$4 \sec ^{2} A=1+\tan ^{2} A$
$5 \operatorname{cosec}^{2} A=1+\cot ^{2} A$
$6 \sin 2 A=2 \sin A \cos A$
$7 \cos 2 A=\cos ^{2} A-\sin ^{2} A$

$$
\begin{aligned}
& =2 \cos ^{2} A-1 \\
& =1-2 \sin ^{2} A
\end{aligned}
$$

$9 \sin (A \pm B)=\sin A \cos B \pm \cos A \sin B$
$10 \cos (A \pm B)=\cos A \cos B \mp \sin A \sin B$
$11 \tan (A \pm B)=\frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
$12 \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$
$13 a^{2}=b^{2}+c^{2}-2 b c \cos A$
14 Area of triangle $=\frac{1}{2} a b \sin C$
$8 \tan 2 A=\frac{2 \tan A}{1-\tan ^{2} A}$

## SECTION A

## [40 marks]

## Answer all questions in this section.

1 Solve the simultaneous equations $x-2 y+6=0$ and $x^{2}+x y-20=0$. Give your answer correct to 3 decimal places.

2 Diagram 1 shows a circle with centre $O$.

$P T Q$ is a tangent to the circle at $T$ and $P Q=O Q=20 \mathrm{~cm}$.
Calculate
(a) the length of the arc $S T R$,
(b) the area of the shaded region.

3 Table 1 shows the frequency distribution of scores of a group of players in a game.

| Score | $0-4$ | $5-9$ | $10-14$ | $15-19$ | $20-24$ | $25-29$ | $30-34$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of players | 2 | 3 | 10 | 20 | $w$ | 6 | 2 |

TABLE 1
It is given that the median of the distribution is 17 .
(a) Calculate the value of $w$.
(b) Hence, calculate the variance of the distribution.

4 (a) If the volume of a cube decreases from $125 \mathrm{~cm}^{3}$ to $124.4 \mathrm{~cm}^{3}$, find the small change in the sides of the cube.
[3 marks]
(b) Given that $f(x)=\frac{3 x+4}{3-x^{2}}$, find the value of $f^{\prime}(2)$.

5 (a) Prove that $\sin 2 x=2 \sin ^{2} x \cot x$.
[2 marks]
(b) Sketch the graph of $y=|2 \sin 2 x|$ for $0 \leq x \leq \pi$. Hence, using the same axes, sketch a suitable straight line to find the number of solutions of the equation $|2 \sin 2 x|$ $=\frac{2 x}{\pi}$ for $0 \leq x \leq \pi$. State the number of solutions.
[6 marks]
(a)


DIAGRAM 2
A piece of wire is cut into 15 parts which are bent to form circles as shown in Diagram 2.
The radius of each circle increases by 3 cm consecutively.
Calculate
(i) the radius of the last circle,
(ii) the area of the last circle.
(b) Diagram 3 shows a rectangular geometric pattern.


DIAGRAM 3

The first rectangle is $A B C D$ and followed by $M B N P$ and so on. The length and width of the next rectangle is half of the length and width of the previous rectangle. Given that $A B=30 \mathrm{~cm}$ and $B C=20 \mathrm{~cm}$. Find the perimeter of the seventh rectangle.
[3 marks]

## SECTION B

## [40 marks]

## Answer four questions from this section.

7 Use graph paper to answer this question.
Table 2 shows the values of two variables $x$ and $y$ which are related by $y=p q^{x+2}$, where $p$ and $q$ are constants.

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ | 25.6 | 125.9 | 640 | 3163 | 15849 | 63096 |

TABLE 2
(a) Convert $y=p q^{x+2}$ to a linear form of $Y=m X+c$.
[2 marks]
(b) Plot $\log _{10} y$ against $(x+2)$ by using a scale of 2 cm to 1 unit on the $Y$-axis and 2 cm to 1 unit on the $X$-axis. Hence, draw the line of best fit.
(c) From the graph in (b), find the value of $p$ and of $q$.
[4 marks]

8 Diagram 4 shows a triangle $O P Q$. The point $R$ lies on $O P$ and the point $S$ lies on $P Q$. The straight line $Q R$ intersects the straight line $O S$ at point $T$.


Given $O P: O R=4: 3, P Q: P S=2: 1, \overrightarrow{O P}=12 \underset{\sim}{x}$ and $\overrightarrow{O Q}=9 \underset{\sim}{y}$.
(a) Express, in terms of $\underline{x}$ and / or $\underline{y}$,
(i) $\overrightarrow{Q R}$,
(ii) $\overrightarrow{O S}$.
[3 marks]
(b) If $\overrightarrow{O T}=h \overrightarrow{O S}$ and $\overrightarrow{Q T}=k \overrightarrow{Q R}$, where $h$ and $k$ are constants, find the values of $h$ and $k$.
[5 marks]
(c) Given that $|\underset{\sim}{x}|=3$ units, $|\underset{\sim}{y}|=5$ units and $\angle P O Q=90^{\circ}$, find $|\overrightarrow{P Q}|$.
[2 marks]

9 (a) In a certain area, 30\% of the trees are rubber trees.
(i) If 8 trees in the area are chosen at random, find the probability that at least two of the trees are rubber trees.
(ii) If the variance of the rubber trees is 315 , find the number of rubber trees in the area.
(b) The masses of the children in the Primary One in the school have a normal distribution with mean 33.5 kg and variance $25 \mathrm{~kg}^{2} .150$ of the children have masses between 30 kg and 36.5 kg . Calculate the total number of children in Primary One in that school.
[5 marks]

10 Solution by scale drawing is not accepted.
In Diagram 5, point $T$ lies on the perpendicular bisector of $A B$.

(a) Find the equation of straight line $A B$.
[2 marks]
(b) A point $P$ moves such that $P A=2 A B$. Find the equation of locus of $P$.
(c) Locus of $P$ intersects the $x$-axis at points $Y$ and $Z$. State the coordinates of $Y$ and $Z$.
(d) Find the $x$-intercept of $C D$.
[2 marks]

11 (a) Given that a curve has a gradient function $p x^{2}+x$ such that $p$ is a constant. $y=6-2 x$ is the equation of tangent to the curve at the point $(2, q)$. Find the value of $p$ and of $q$.
[3 marks]
(b) Diagram 6 shows the curve $y=(x-3)^{2}$ and the straight line $y=2 x+2$ intersect at point (1, 4).


Calculate
(i) the area of the shaded region,
(ii) the volume of revolution, in terms of $\pi$, when the region bounded by the curve, the $x$-axis, the $y$-axis and the straight line $x=2$ is revolved through $360^{\circ}$ about the $x$-axis.
[3 marks]

## SECTION C

## [20 marks]

Answer two questions from this section.
12 A particle moves along a straight line and passes through a fixed point $O$. Its velocity, $v$ $\mathrm{ms}^{-1}$, is given by $v=t^{2}-6 t+5$, where $t$ is the time, in seconds, after passing through $O$. (Assume motion to the right is positive).
Find
(a) the initial velocity, in $\mathrm{ms}^{-1}$,
(b) the minimum velocity, in $\mathrm{ms}^{-1}$,
(c) the range of values of $t$ at which the particles moves to the left,
(d) the total distance, in m , travelled by the particle in the first 5 seconds.
[4 marks]

13 In Diagram 7, $A B C$ is a triangle. $B M C$ and $A M$ are straight lines.

(a) Calculate
(i) $\angle A M B$,
(ii) the area, in $\mathrm{cm}^{2}$, of triangle $A B C$.
(b) A new triangle $A^{\prime} B^{\prime} M^{\prime}$ is formed with $A^{\prime} B^{\prime}=A B, B^{\prime} M^{\prime}=B M$ and $\angle B^{\prime} A^{\prime} M^{\prime}=\angle B A M$, find the length of $A^{\prime} M^{\prime}$.
[3 marks]

14 Use the graph paper provided to answer this question.
A factory produces two types of school bags $\boldsymbol{M}$ and $\boldsymbol{N}$ using two types of machines $\boldsymbol{A}$ and $\boldsymbol{B}$. Given that machine $\boldsymbol{A}$ requires 20 minutes to produce a bag $\boldsymbol{M}$ and 30 minutes to produce a bag $\boldsymbol{N}$ while machine $\boldsymbol{B}$ requires 25 minutes to produce a bag $\boldsymbol{M}$ and 40 minutes to produce a bag $N$. The machines produce $x$ units of $\boldsymbol{M}$ and $y$ units of $\boldsymbol{N}$ in a particular day according to the following conditions.

I : Machine $\boldsymbol{A}$ is operated for not more than 8 hours.
II : Machine $\boldsymbol{B}$ is operated for at least 4 hours.
III : The number of units of bag $\boldsymbol{M}$ produced is not more than twice the number of units of bag $N$.
(a) Write the three inequalities for the above conditions.
[3 marks]
(b) Using a scale of 2 cm to 2 units for both axes, construct and shade the region $\boldsymbol{R}$ which satisfies all the above conditions.
[3 marks]
(c) Use the graph constructed in 14 (b), to find
(i) the maximum number of units of bag $\boldsymbol{M}$ that can be produced if the factory produces 12 units of bag $\boldsymbol{N}$.
(ii) the maximum profit obtained if the profit from one unit of bag $\boldsymbol{M}$ and bag $\boldsymbol{N}$ are RM 18 and RM 20 respectively.

15 Table 3 shows the price indices and percentage of usage of four components, $P, Q, R$ and $S$, which are the number of parts in the making of an electronic device.

| Item | Price index for the year 2000 based on <br> the year 1997 | Percentage of usage (\%) |
| :---: | :---: | :---: |
| $P$ | 125 | 20 |
| $Q$ | 140 | 10 |
| $R$ | $x$ | 30 |
| $S$ | 110 | 40 |

TABLE 3
(a) Calculate
(i) the price of $Q$ in the year 1997 if its price in the year 2000 is RM 50.40,
(ii) the price index of $P$ in the year 2000 based on the year 1994 if its price index in the year 1997 based on the year 1994 is 120 .
(b) The composite index number for the cost of production in the year 2000 based on the year 1997 is 122 . Calculate
(i) the value of $x$,
(ii) the price of an electronic device in the year 1997 if the corresponding price in the year 2000 was RM 288.

## END OF QUESTION PAPER

